GeoSpec

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Lama

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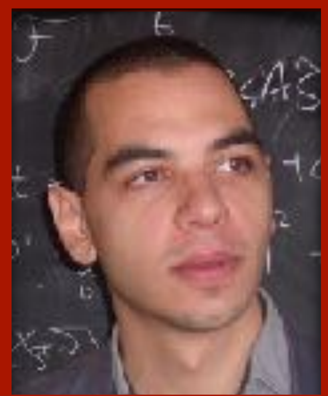


Lama

Fundamental Mathematics and Numerical Tools

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Objectives of the project

- Take benefit of Grenoble's potential which has a strong expertise both in pure mathematics and computer science.
- We plan to develop a new synergy at a national and international level around young researchers of several disciplines. More than new theorems, we would like to introduce new highlights of difficult and important mathematical questions.

Dynamic Complerxity : Birkhoff's sections

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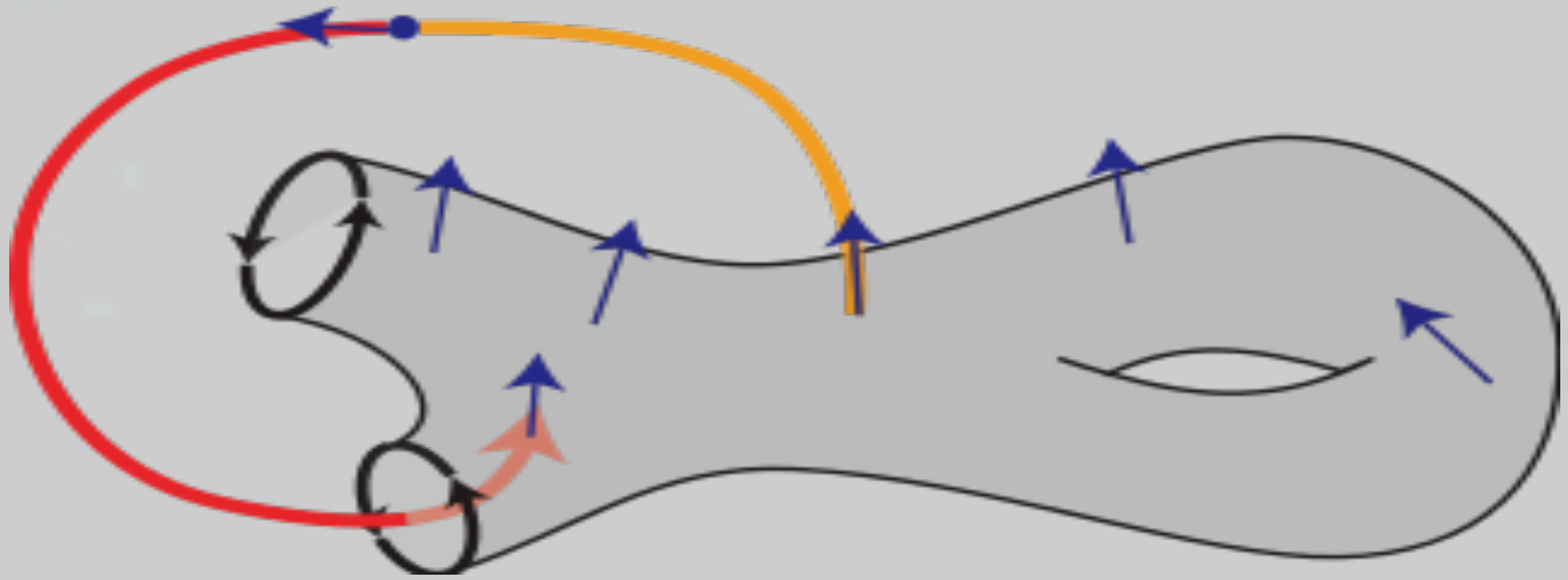
Section associated to a vector field



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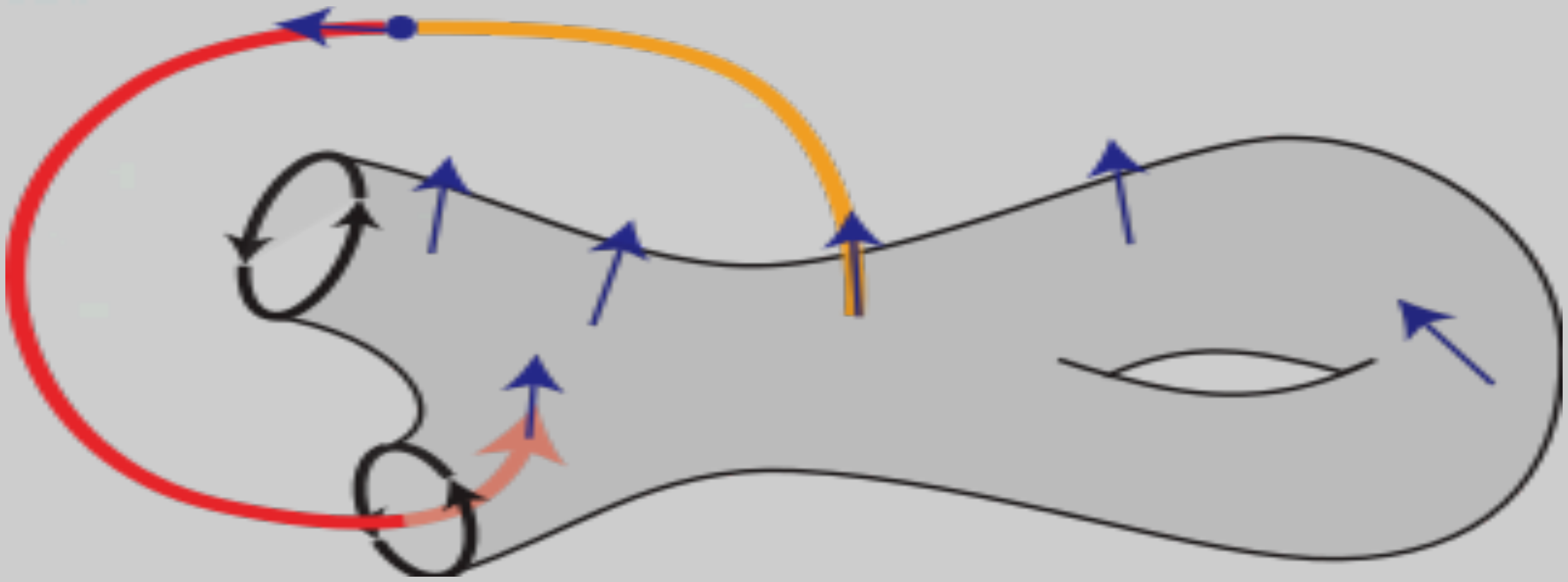

Understand the complexity of a vector field by a section

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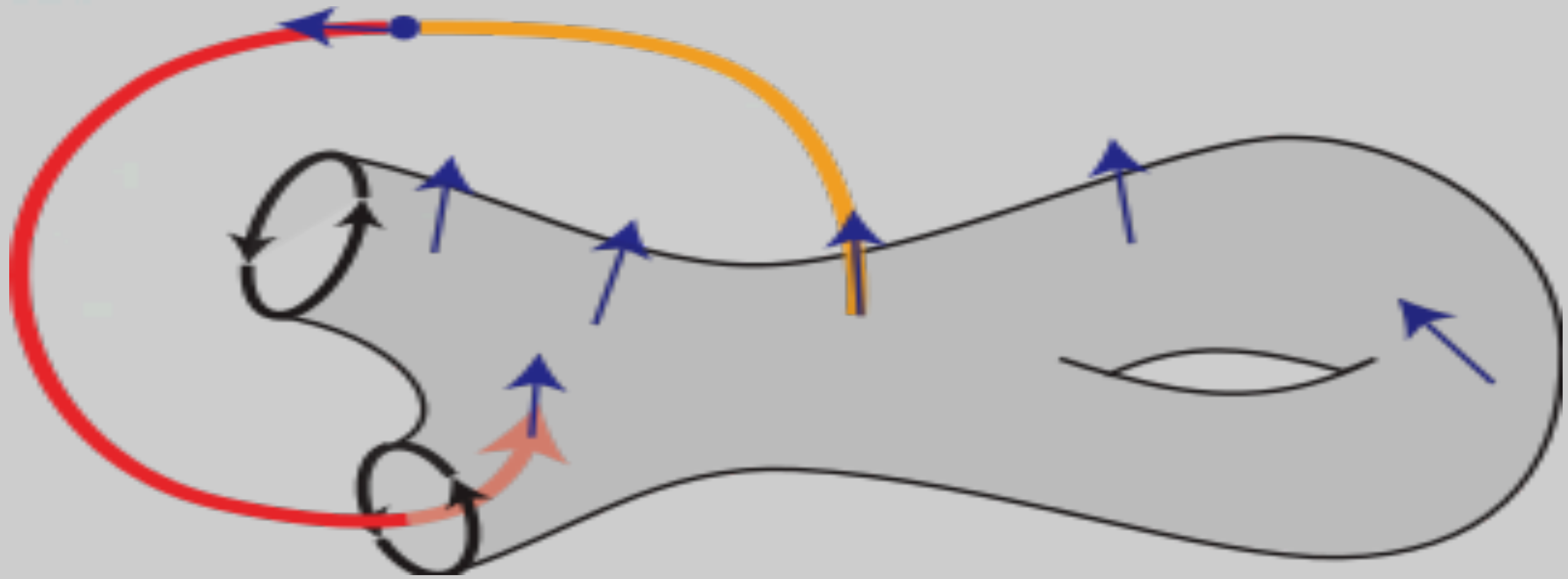

Understand the complexity of a vector field by a section

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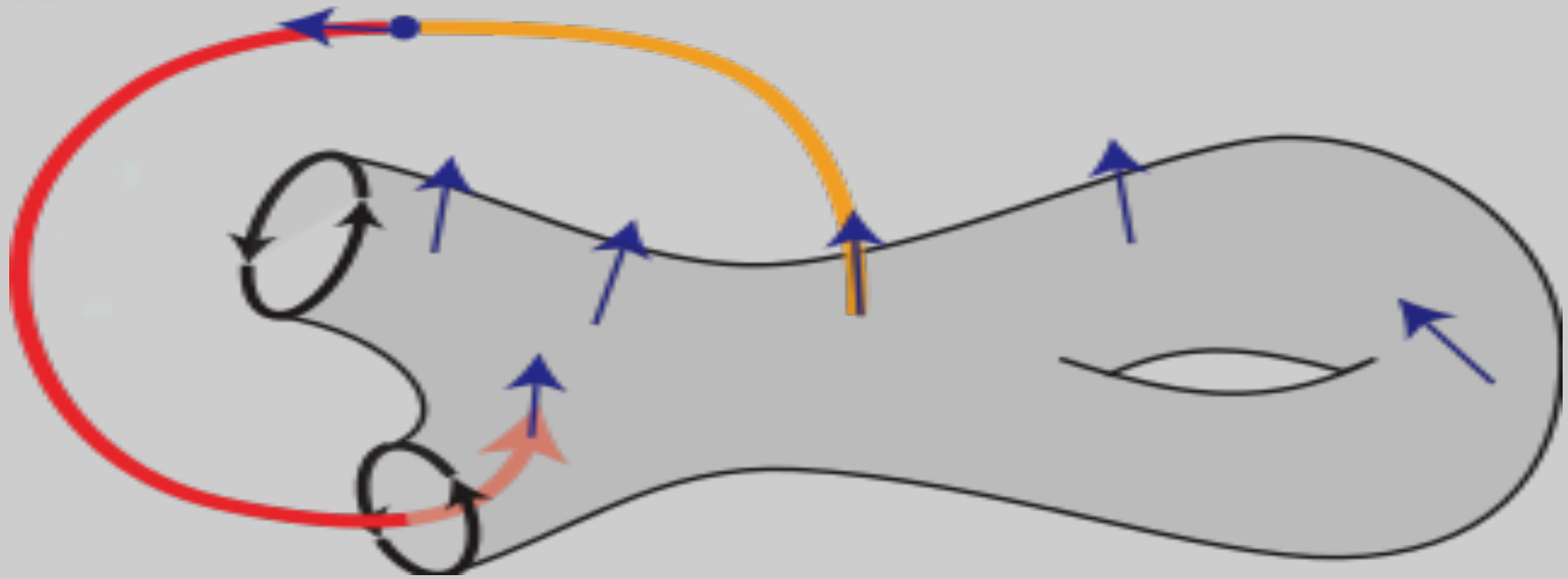

Understand the complexity of a vector field by a section

- Specialists of differential geometry work with *classes* of surfaces
- The non constructive Thurston theorem ensures the existence of simple representations for flat surfaces in the Hyperbolic context. How to identify such representations?

Optimization among metrics

$$\text{pp } v \in T_x S, \;\; \lim_{n\to\infty} \frac{1}{n} \log ||D_x f^n(v)|| = \log(\lambda)$$

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- Linearization, projection //, non smooth and large scale algorithm.

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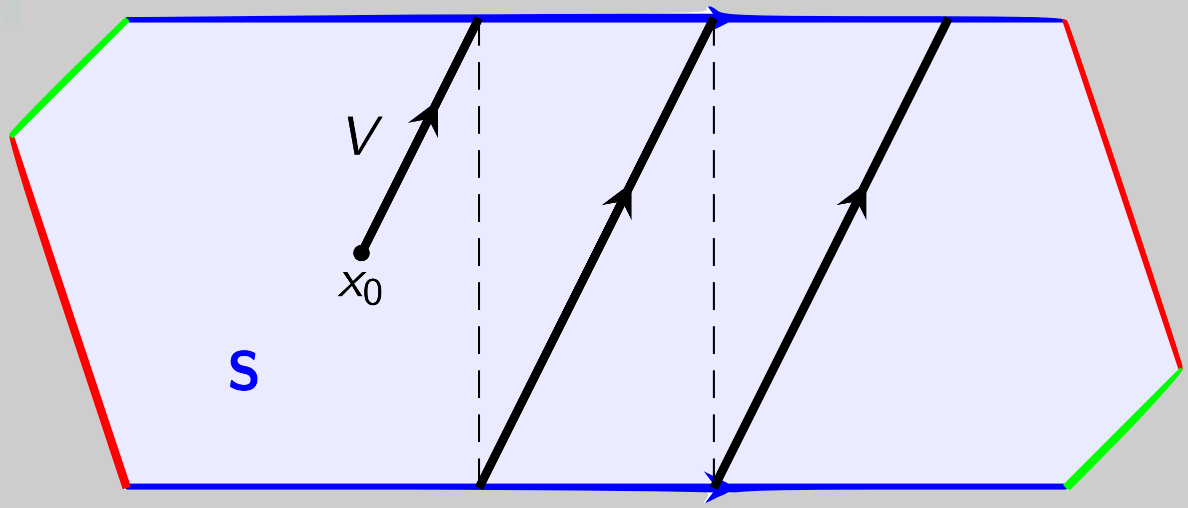

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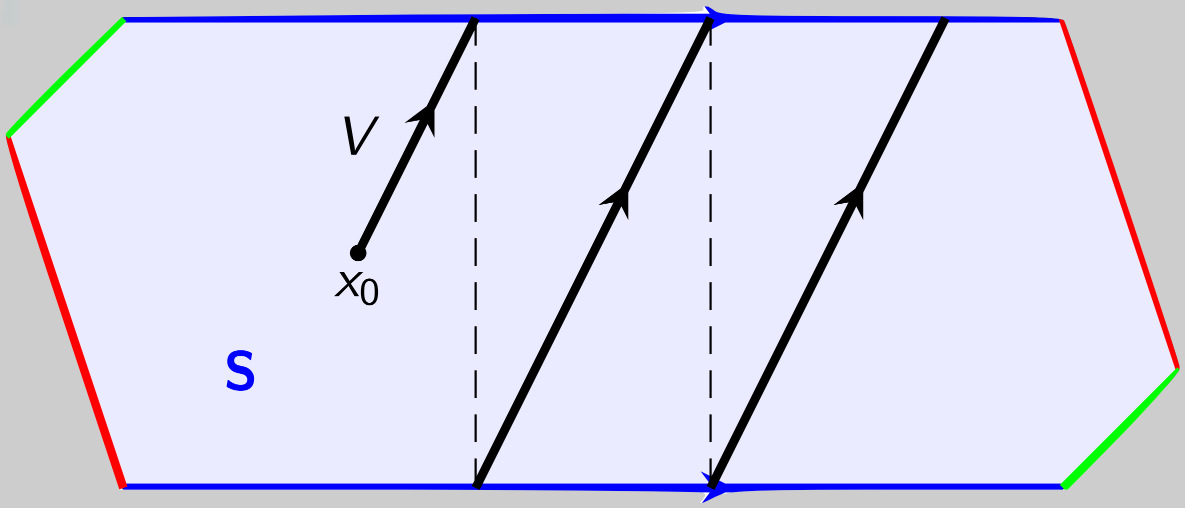

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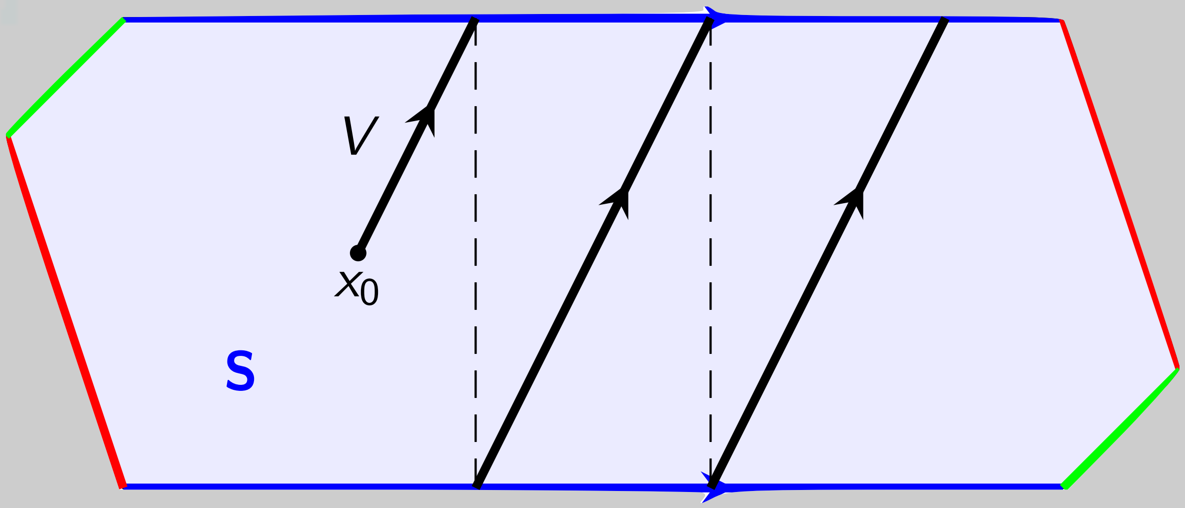

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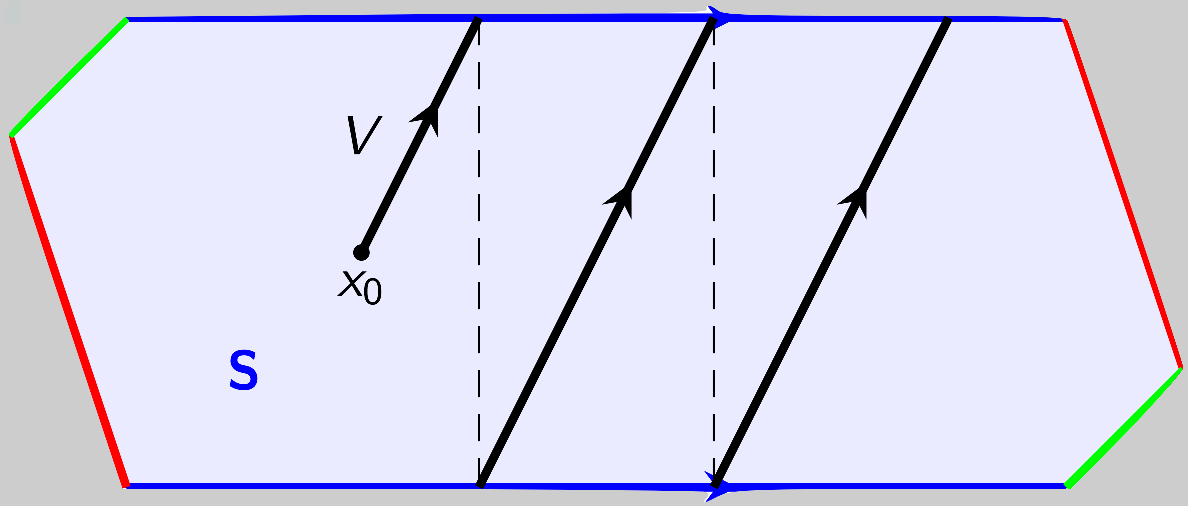

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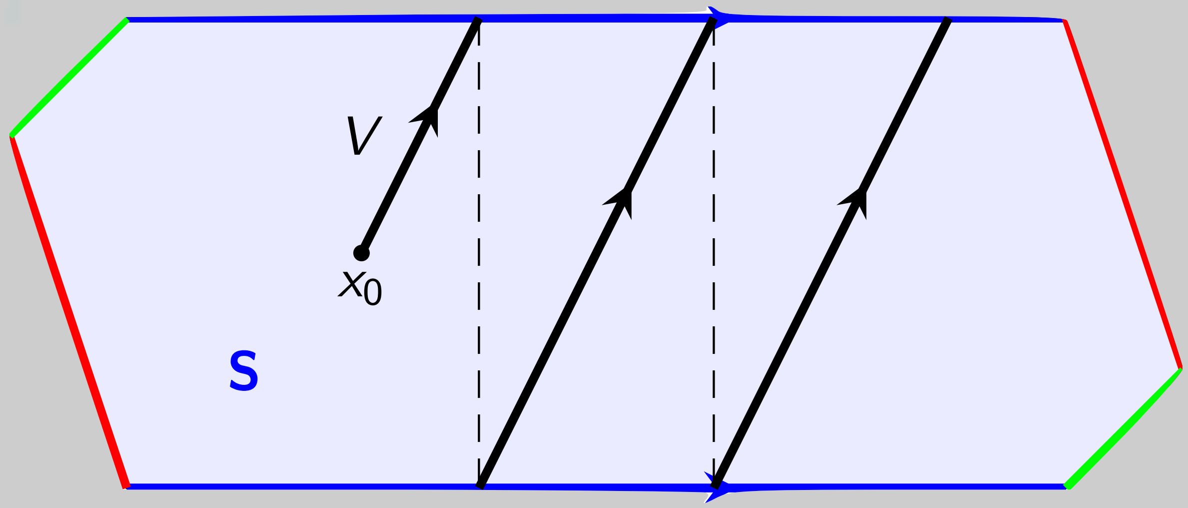

Identify the non trivial shortest geodesic ?

- Combinatorial complexity: all the points/directions, Galois's team
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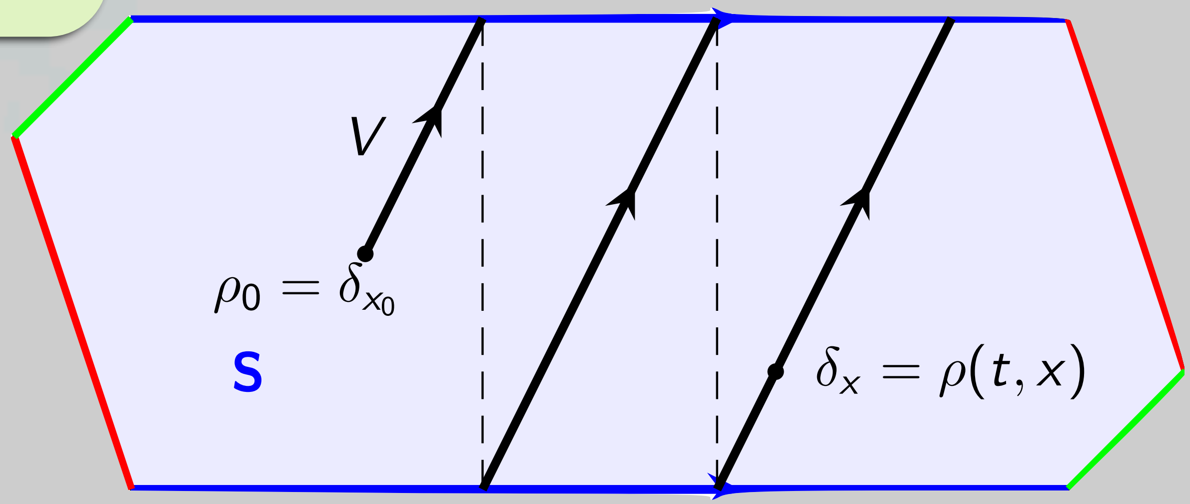
- Curse of dimension

A PDE approach



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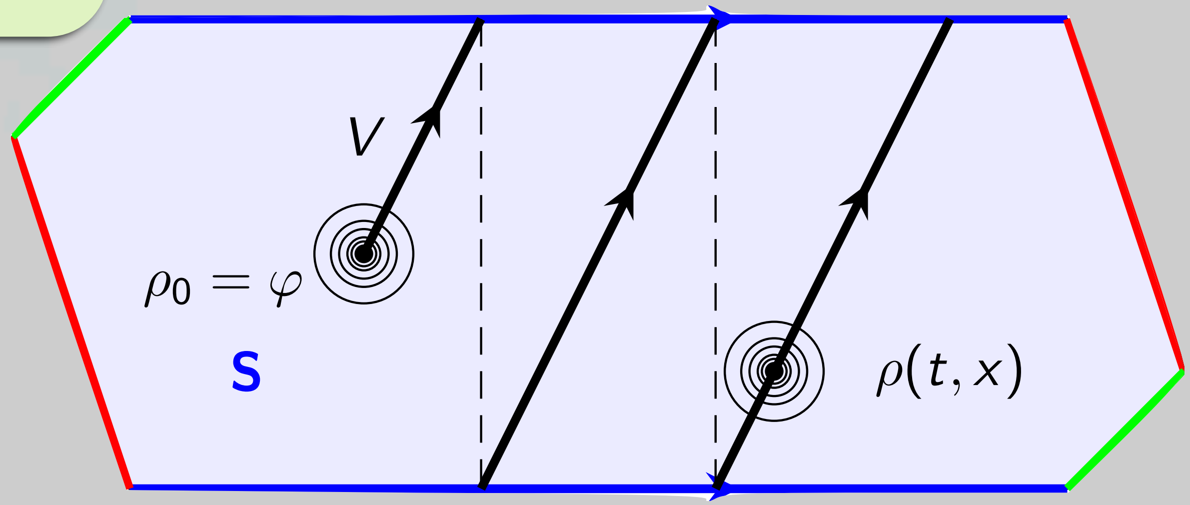

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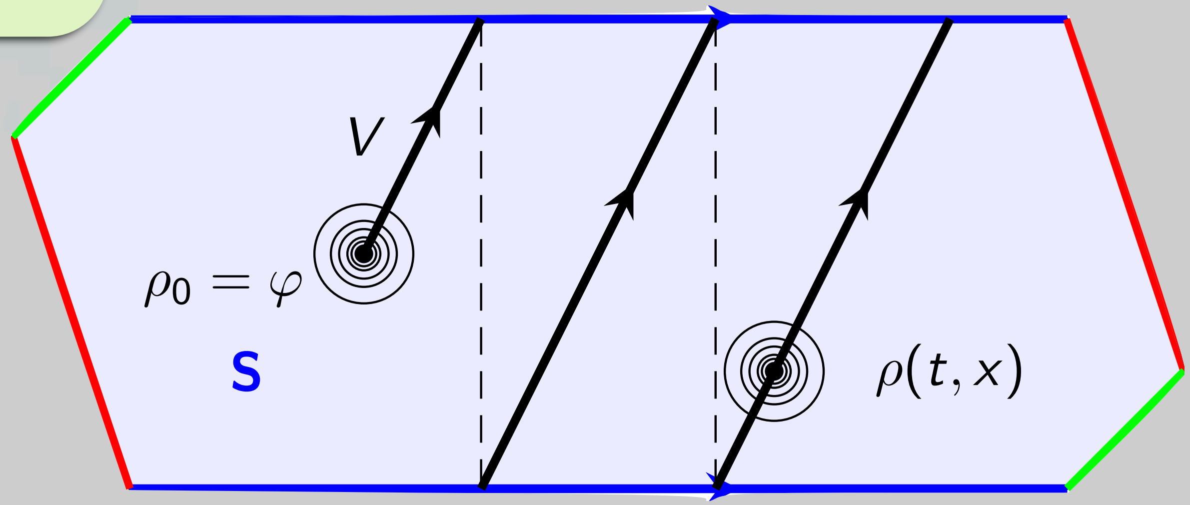

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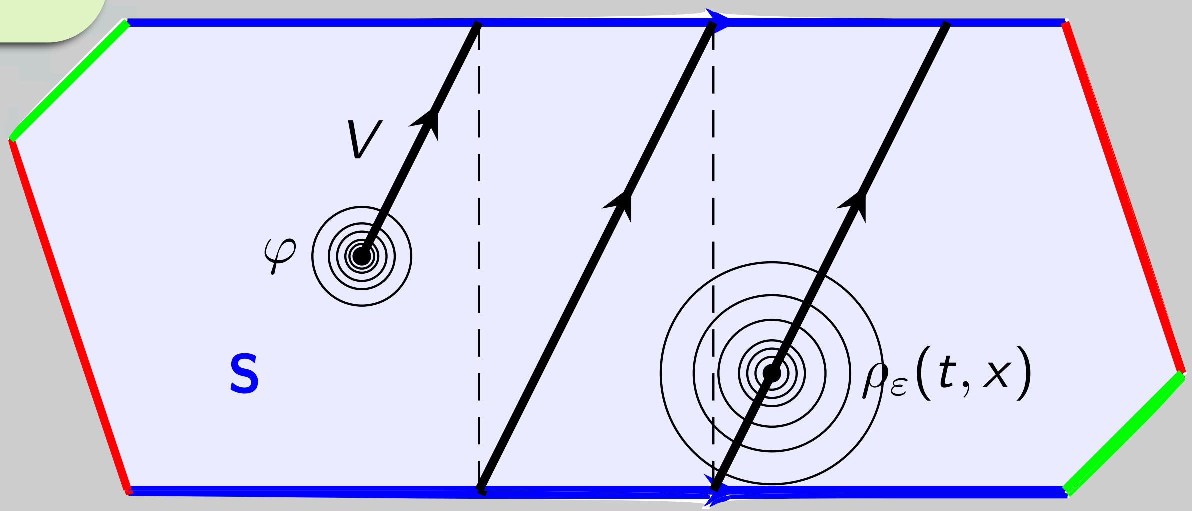

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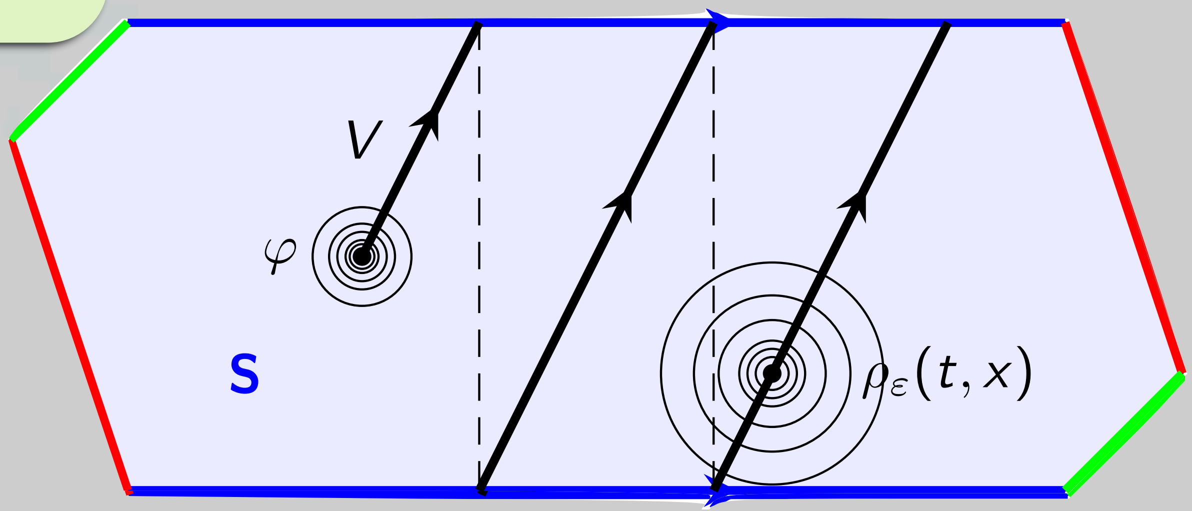

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The speed of diffusion of ρ_ϵ is represented by the eigenvalue λ_ϵ

$$-\epsilon \Delta u_\epsilon + V \cdot \nabla u_\epsilon = \lambda_\epsilon u_\epsilon, \quad \int_S u_\epsilon = 0, \quad \int_S u_\epsilon^2 = 1.$$

The less diffused function is the eigenfunction u_ϵ.

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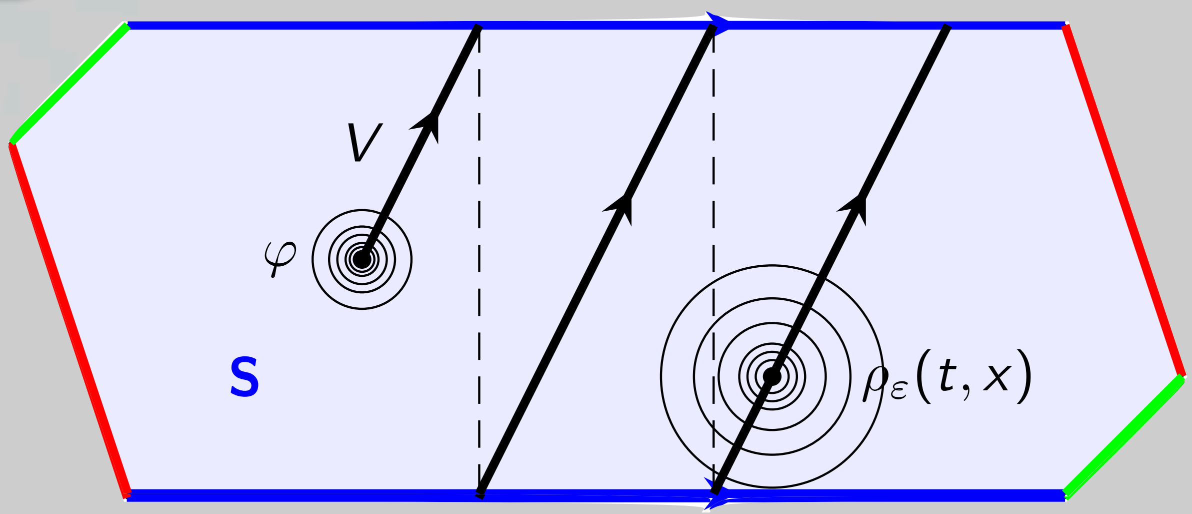

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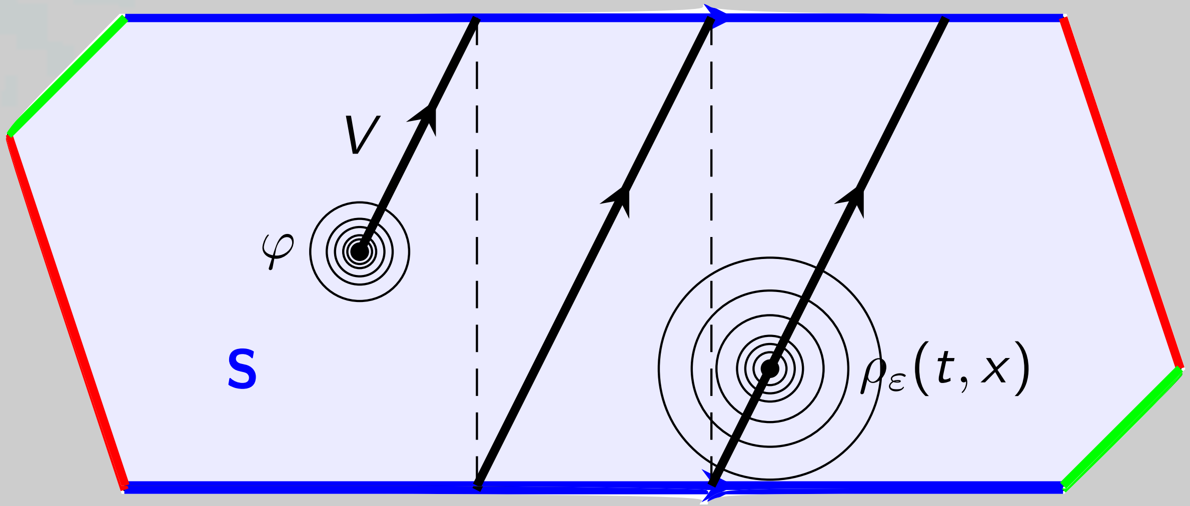

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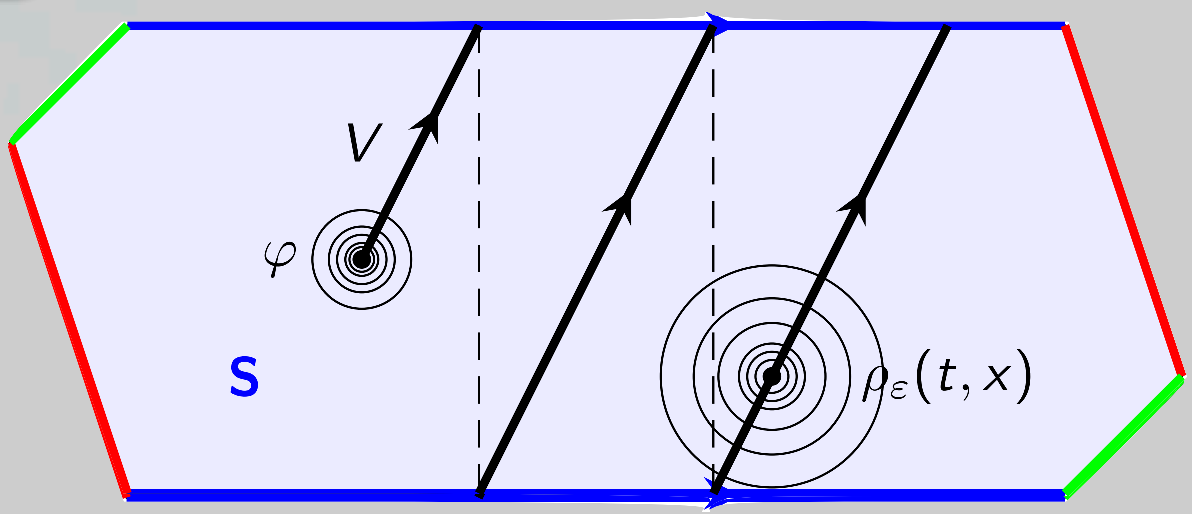

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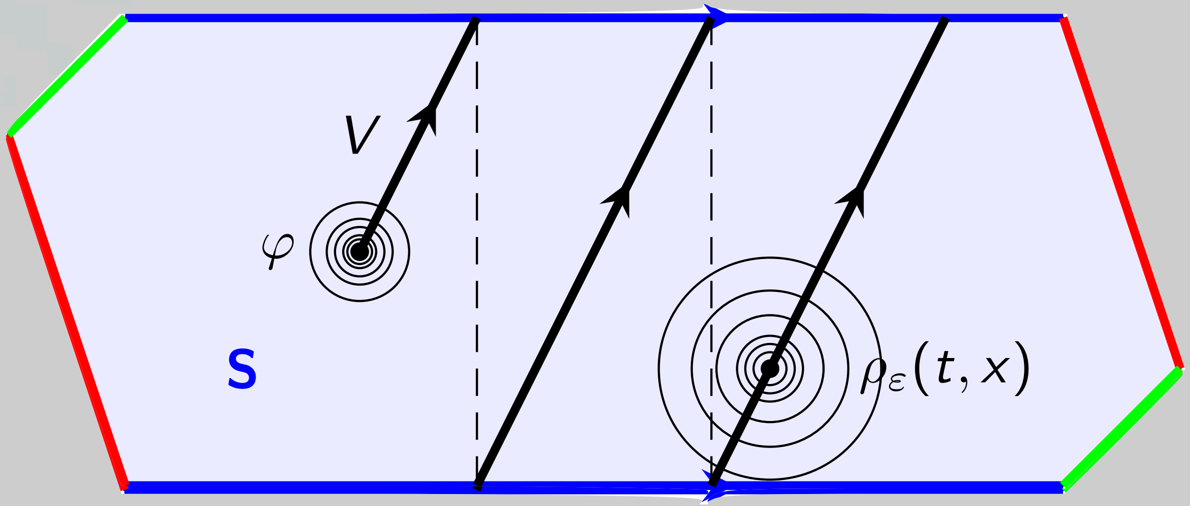

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- Non convex problem: relaxation by considering small parameters?

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Flat Torus and distances: Nash's Isometric problem

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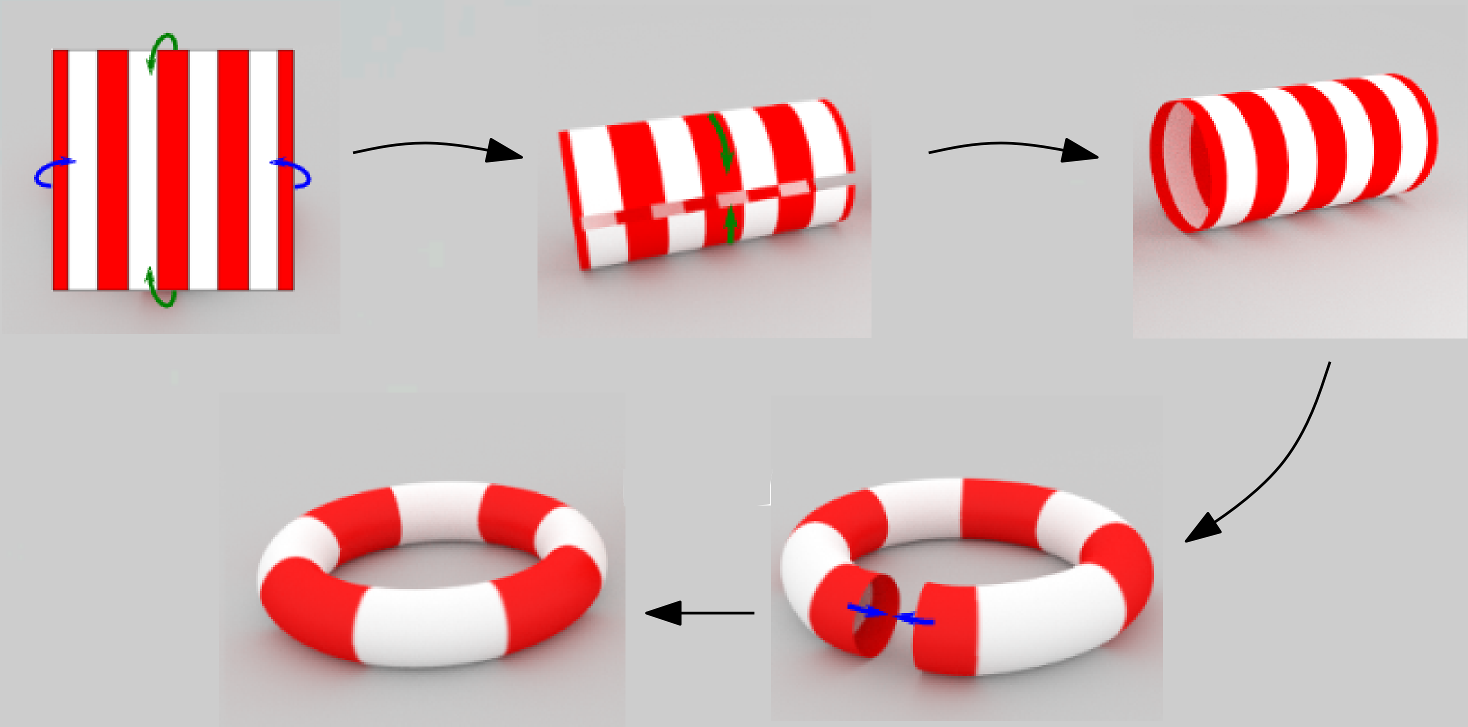

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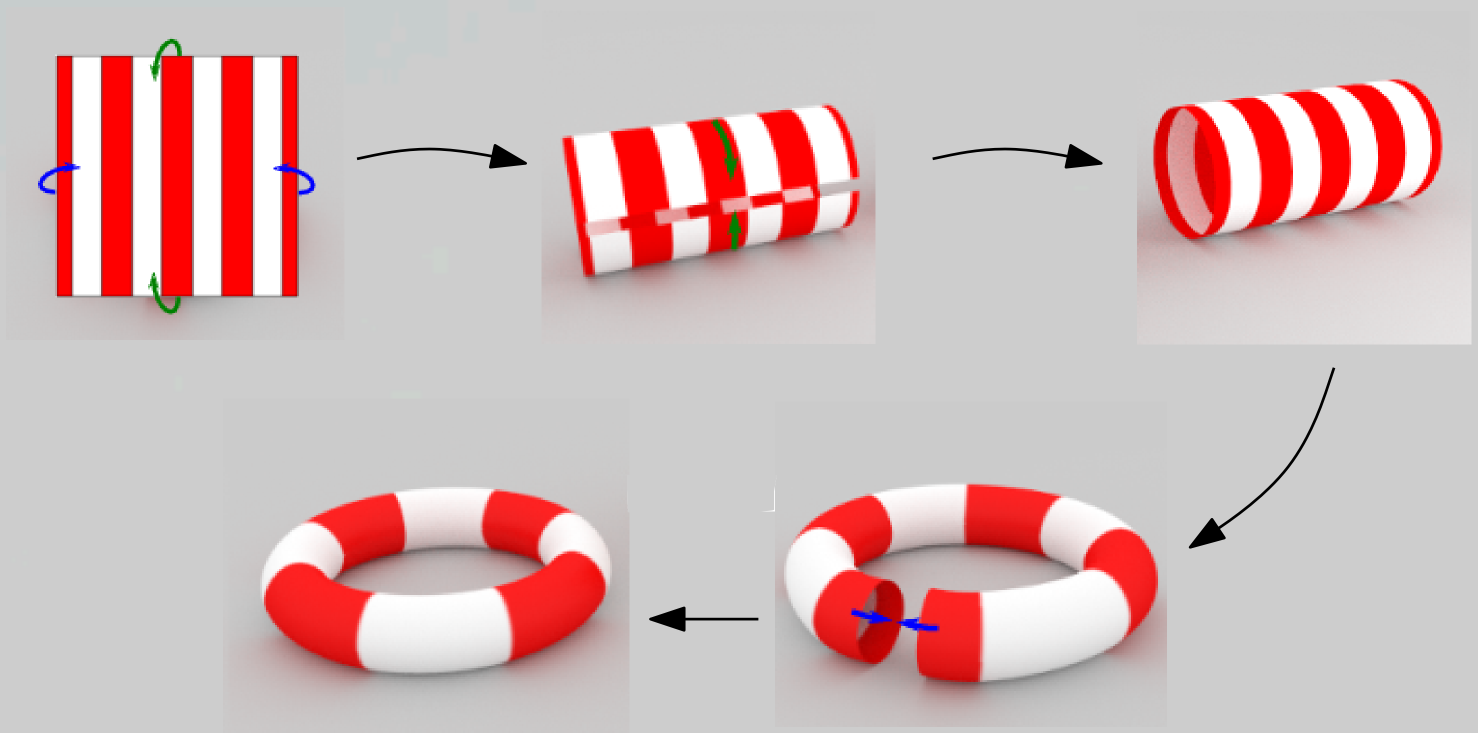

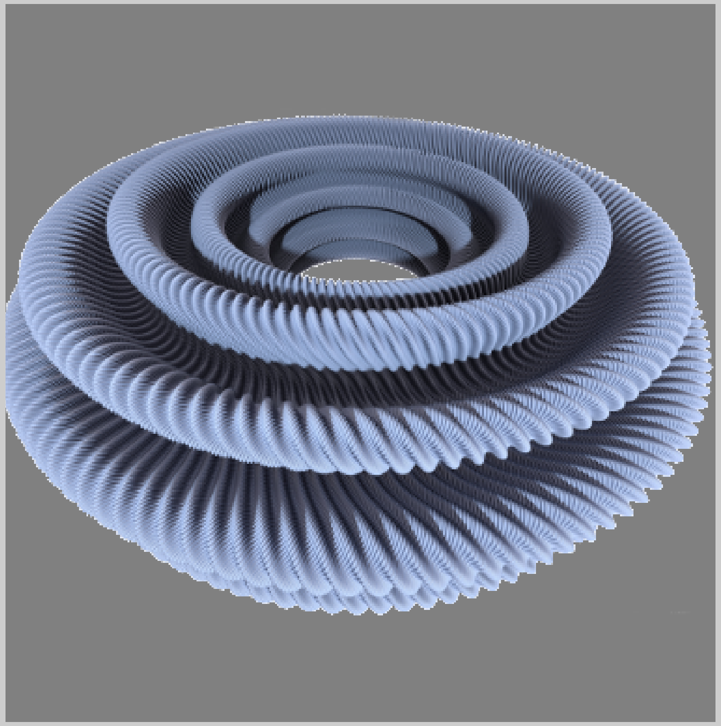

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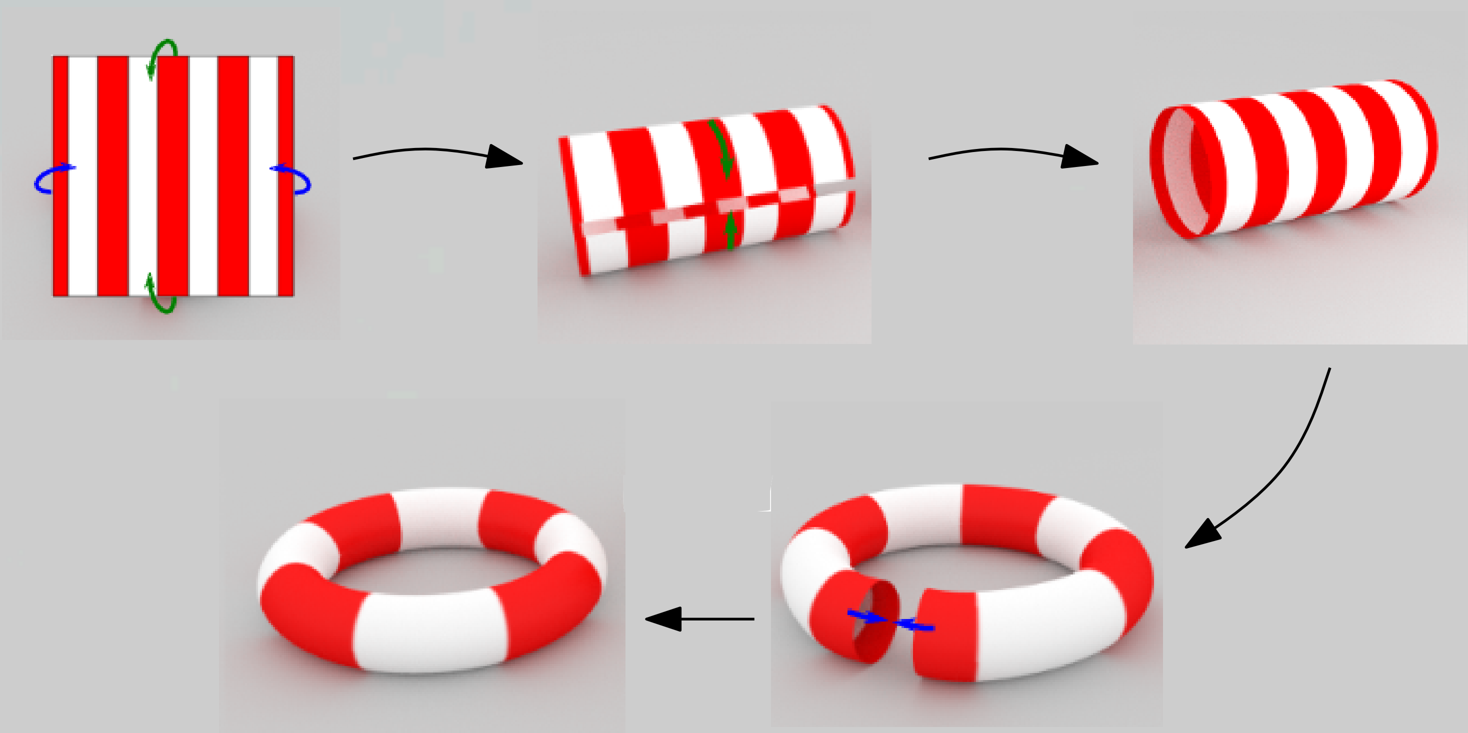

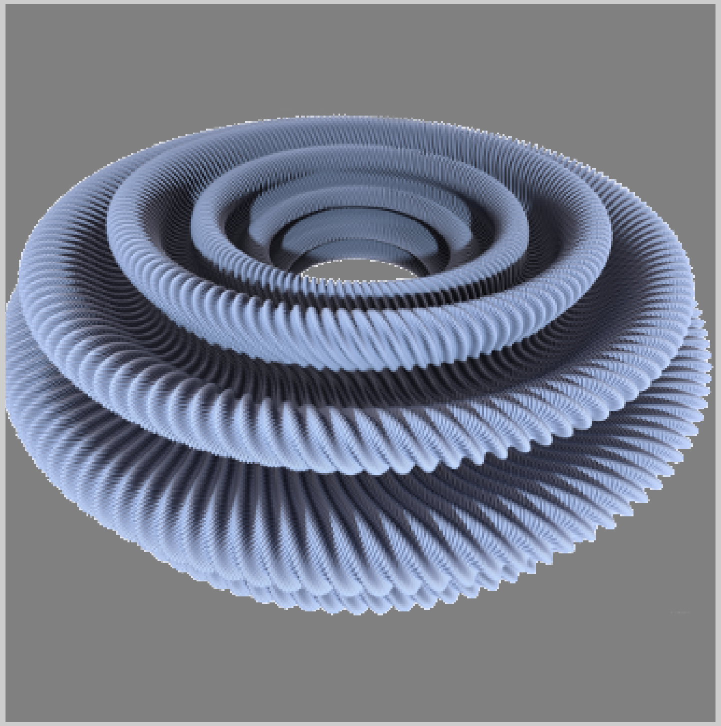

Canonical embeding, an alternative to Gromov's approach

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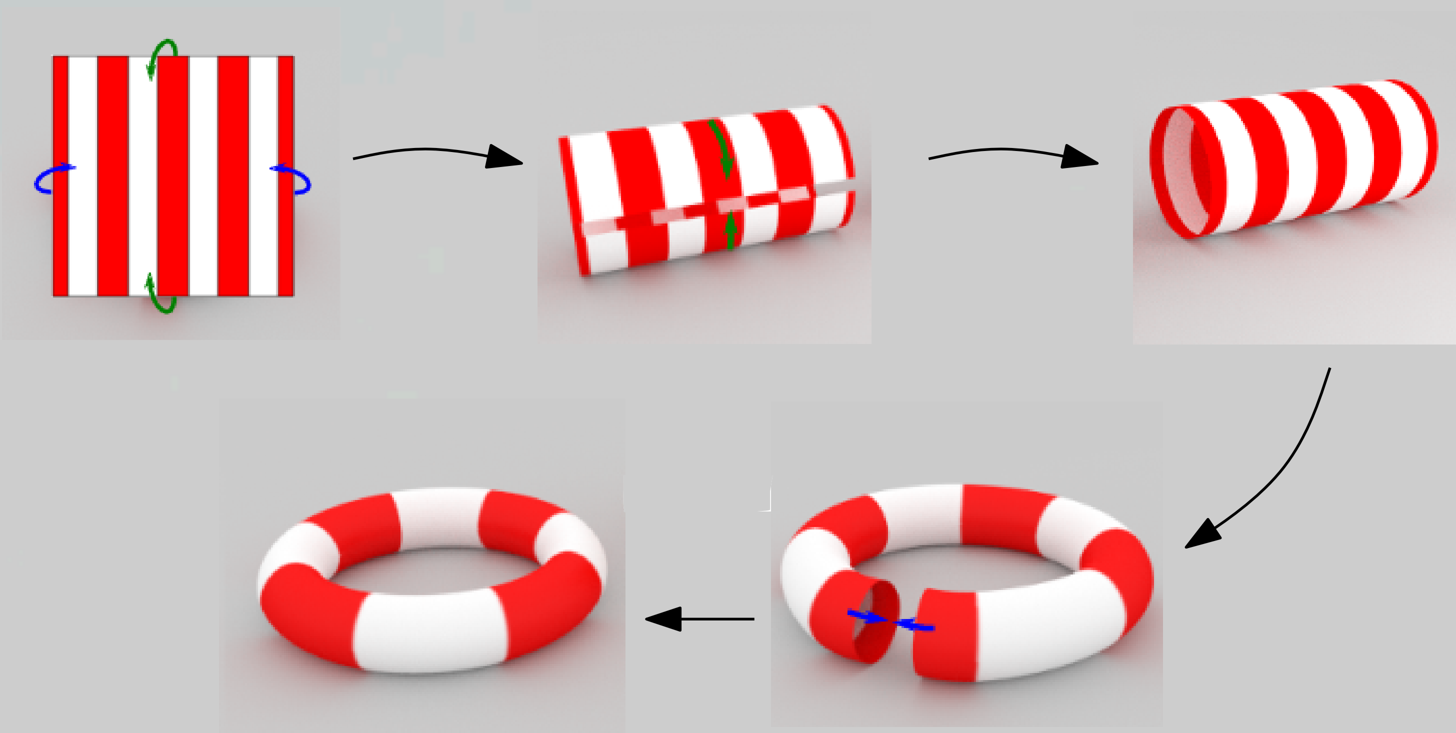

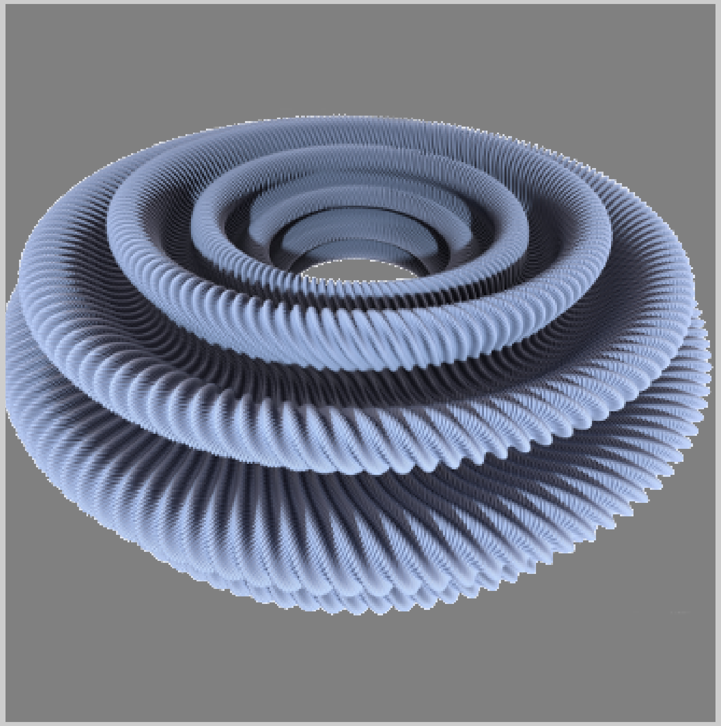

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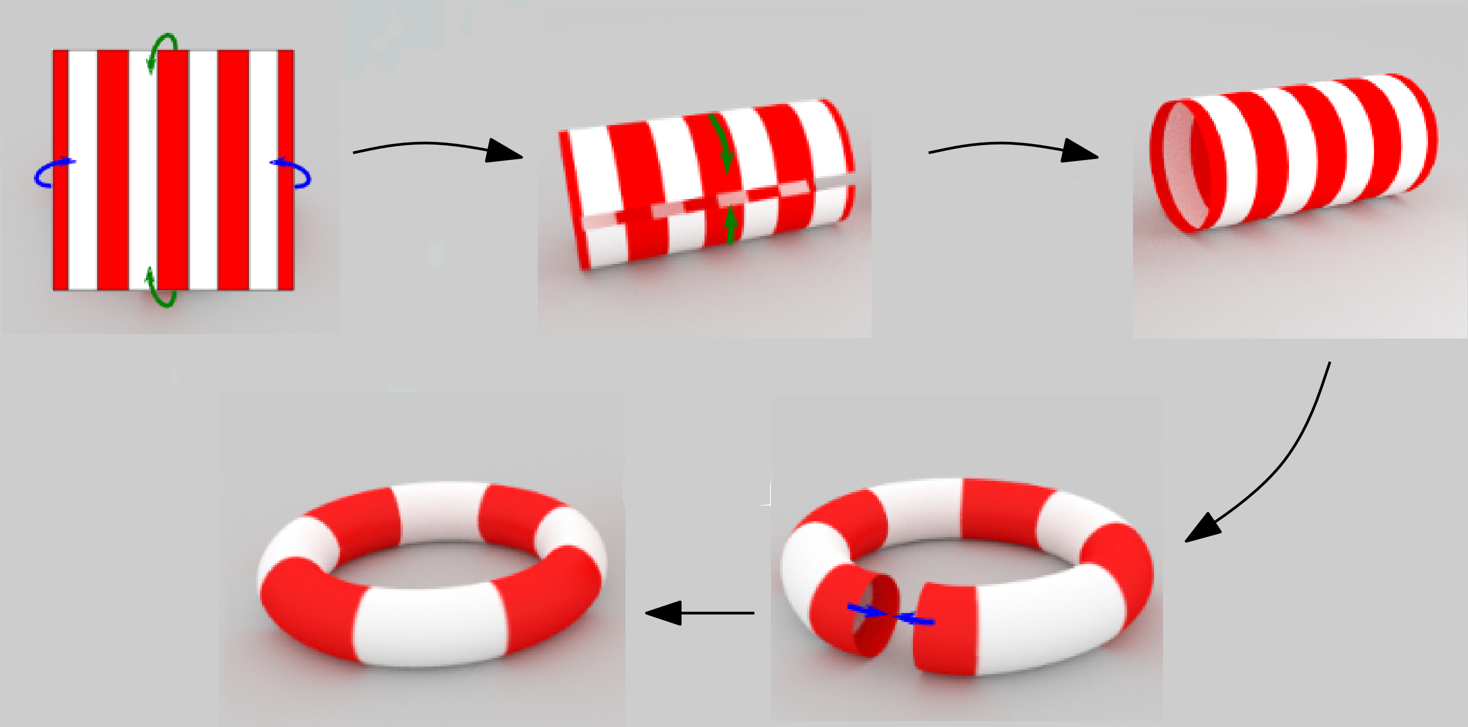

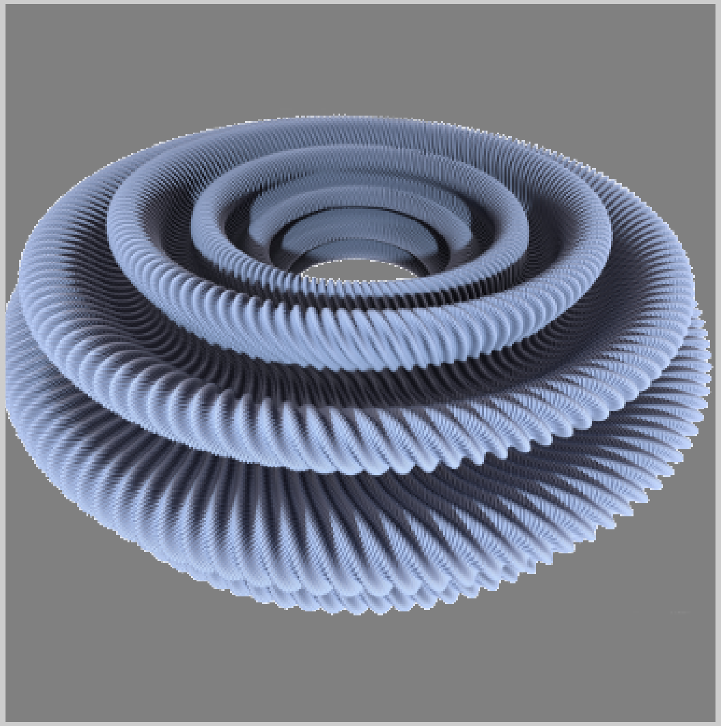

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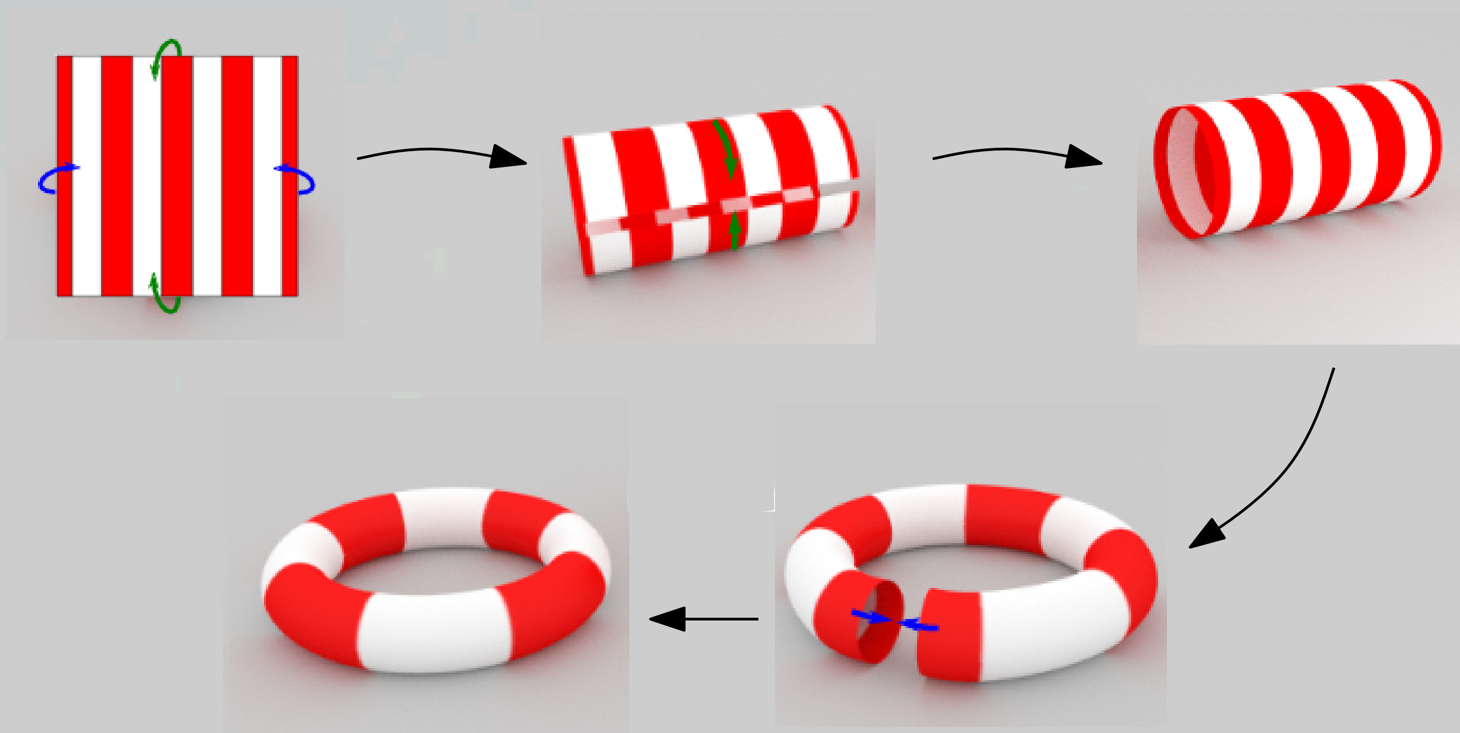

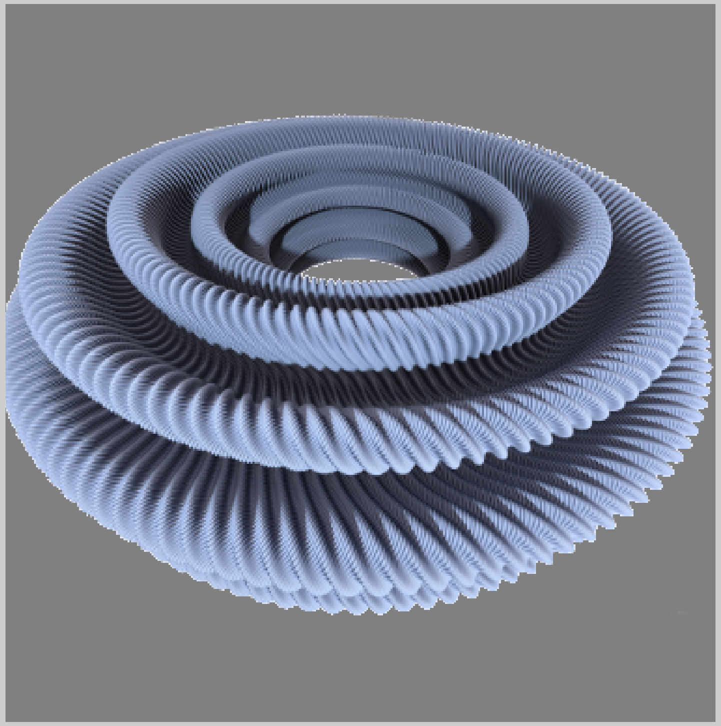

Canonical embeding, an alternative to Gromov’s approach

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- Generalization to genus > 1
- Fractal behavior has to be expected, quantitative estimate

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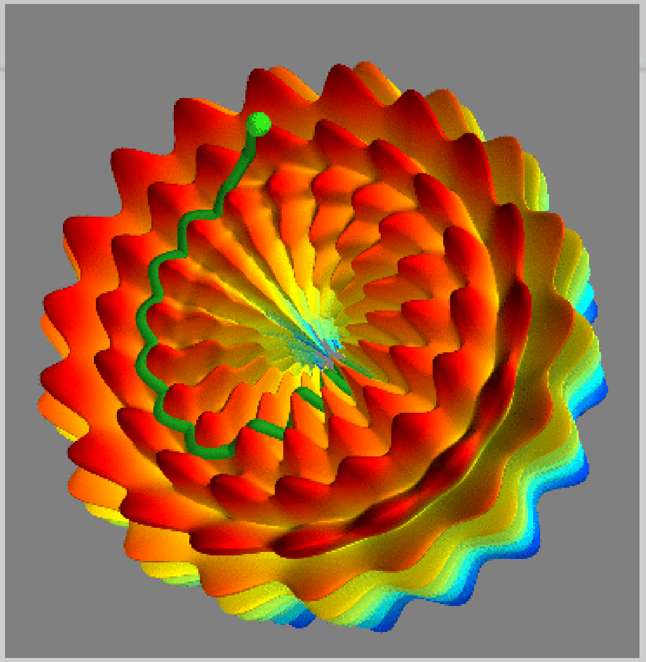

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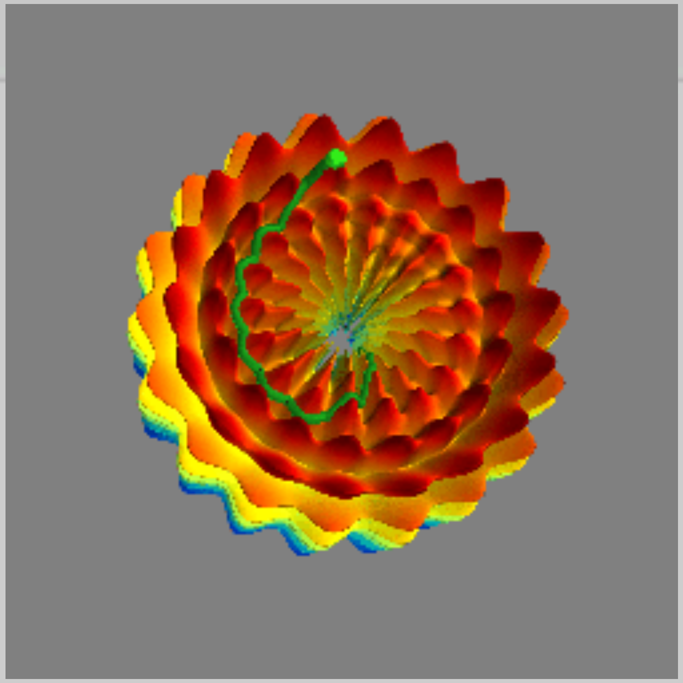

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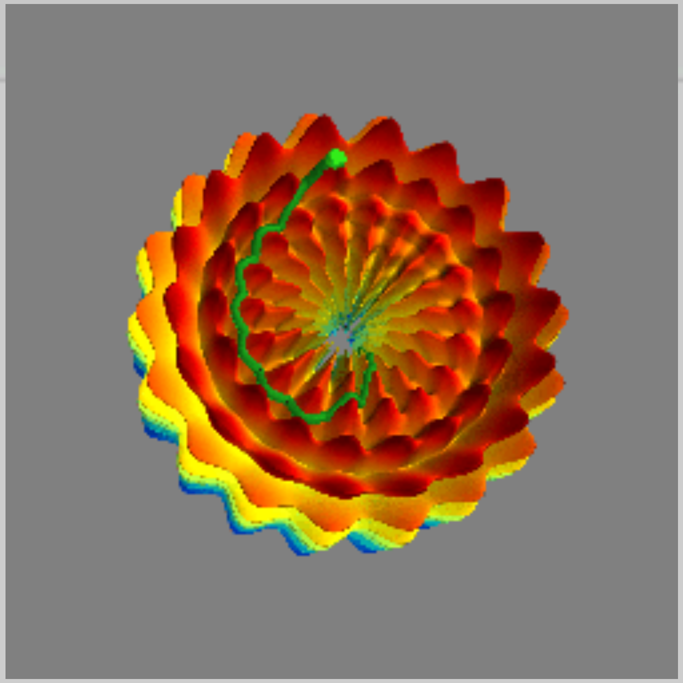

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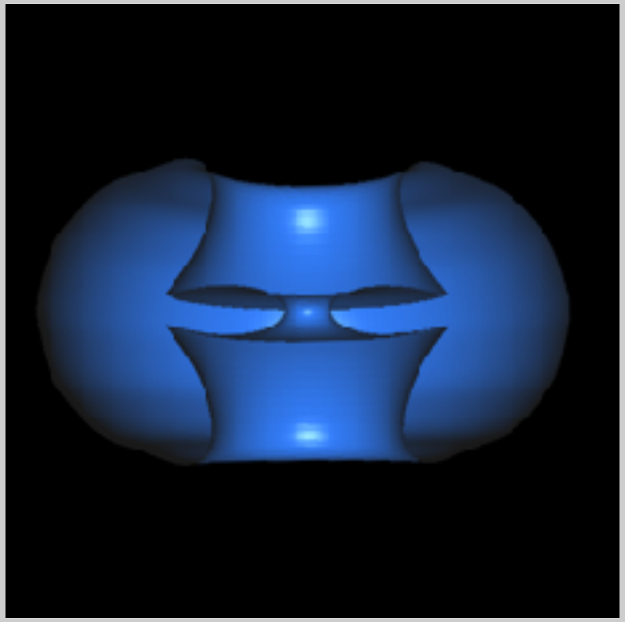

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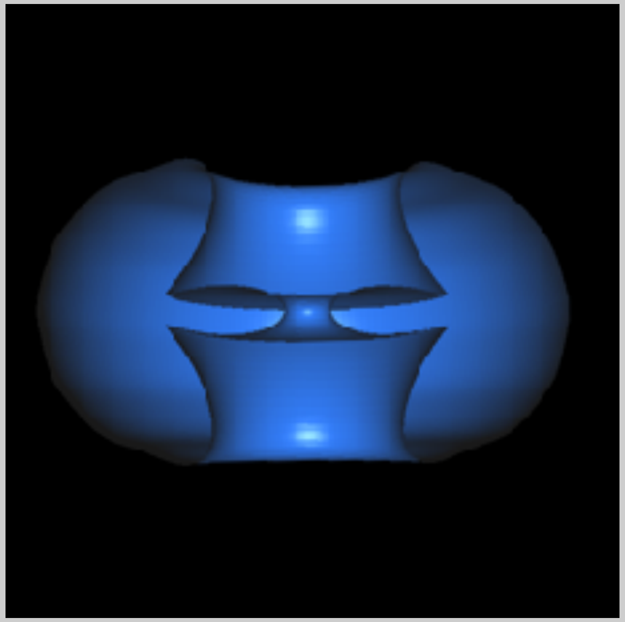

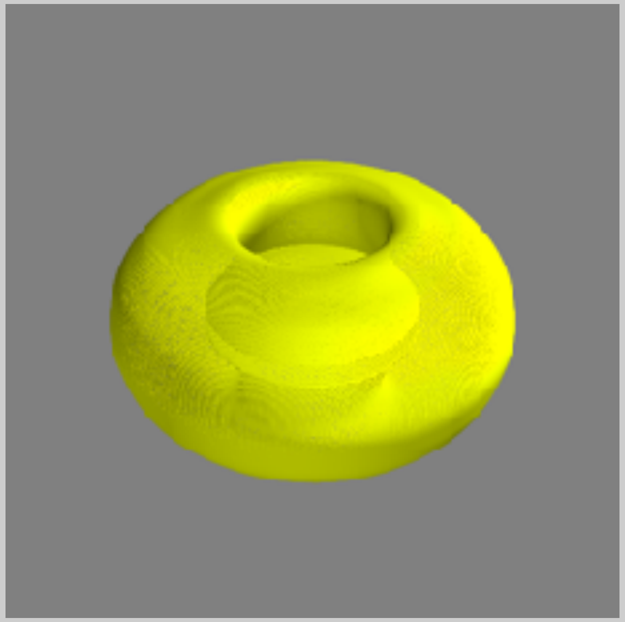

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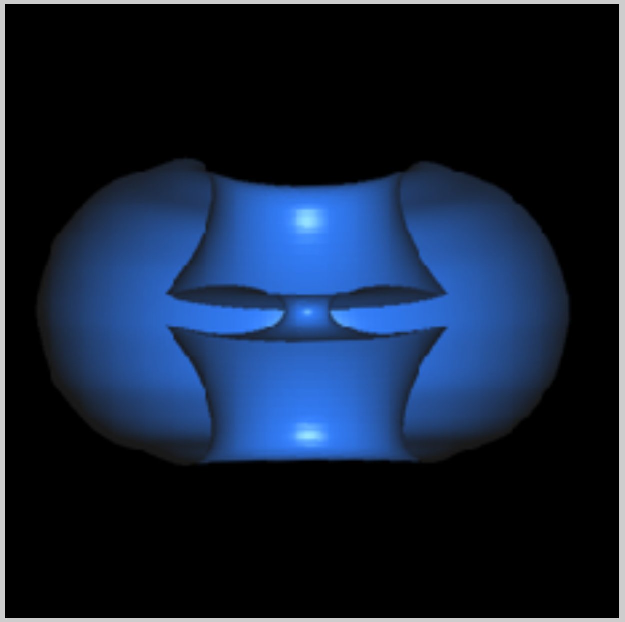
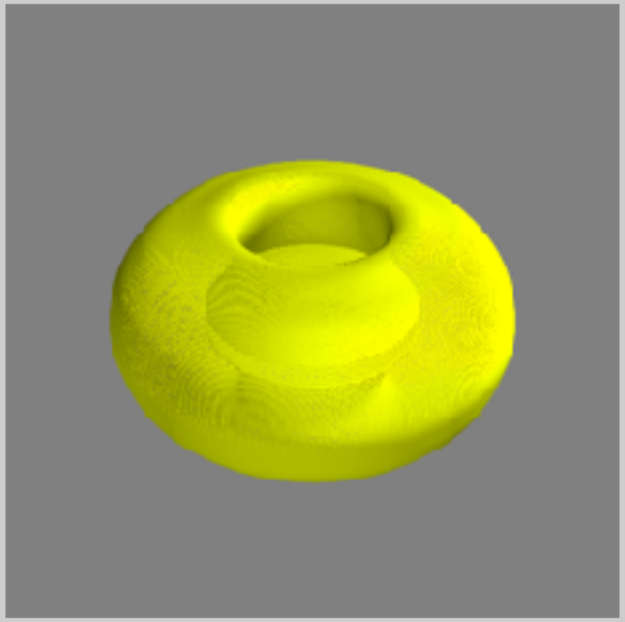
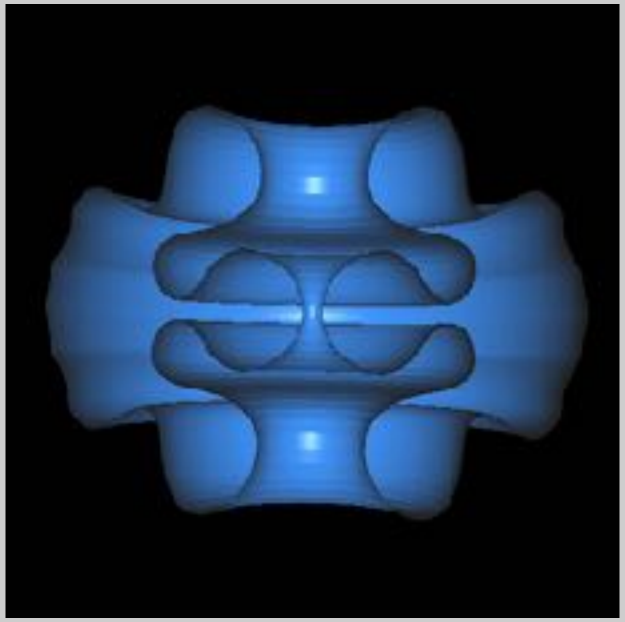

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