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Blind source separation and electroencephalography analysis
a geometrical approach

Persyval days 2017

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Context

Our approach: geometrical modeling of the problem

Numerical experiment

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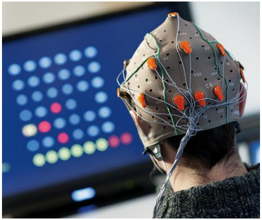
Electroencephalography (EEG)

- recording of the electrical activity on the scalp resulting from the electrical activity of the brain

- applications:
  - brain research
  - diagnosis - epilepsy, sleep disorders,...
  - neurofeedback - modulate its own brain activity
  - brain computer interface - video games, assistance to disabled persons

- interests:
  - low cost
  - non-invasive
  - very good temporal resolution

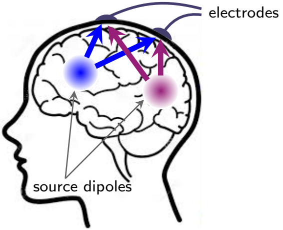
    well capture the dynamics of brain activity



Electroencephalography (EEG)

- recorded activity generated by electrical source dipoles inside the brain

simultaneous activation of colons of neurons



- source signals are mixed while propagating through the brain, skull and scalp [Nunez and Srinivasan, 2006]
- recorded signals $x(t) \in \mathbb{R}^n$ follow the mixing process:

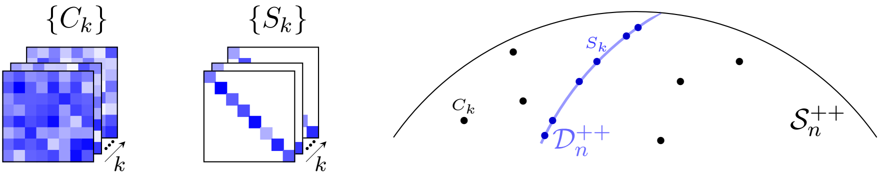
$$x(t) = As(t),$$

  - $s(t) \in \mathbb{R}^p$, source signals
  - $A \in \mathbb{R}^{n \times p}$, mixing matrix

Blind source separation (BSS)

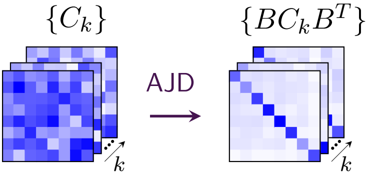
- retrieve the source signals $s(t)$ and the mixing process A from the observations $x(t)$ [Comon and Jutten, 2010]

  only assume that source signals are statistically independent
- use K matrices C_k containing the statistics of $x(t)$:
  - element i, j: statistical link between electrodes i and j
  - in $\mathcal{S}_n^{++}$, set of symmetric positive definite (SPD) matrices
- matrices S_k containing the statistics of $s(t)$ are diagonal



Approximate joint diagonalization (AJD)

- Given $\{C_k\}$, find an invertible matrix $B \in \mathbb{R}^{n \times n}$ such that BC_kB^T are as much diagonal as possible
- estimated source signals are $\tilde{s}(t) = Bx(t)$
- for $K > 2$, no closed form solution - iterative optimization algorithm



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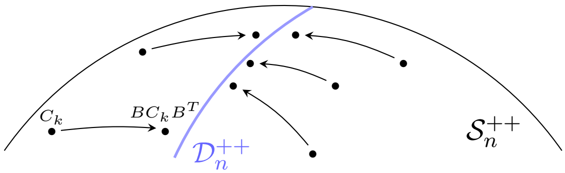
Approximate joint diagonalization (AJD)

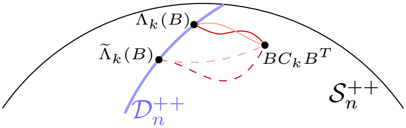
- from a geometrical point of view:



- we want change the basis in order to get the matrices C_k as "close" as possible to $\mathcal{D}_n^{++}$

we need the notion of "distance" of a matrix on $\mathcal{S}_n^{++}$ to the subset $\mathcal{D}_n^{++}$

Approximate joint diagonalization (AJD)



- "distance" from BC_kB^T to $\mathcal{D}_n^{++}$:
  - a divergence $d(\cdot,\cdot)$ on $\mathcal{S}_n^{++}$
    similar to a distance, less properties
  - a diagonal matrix $\Lambda_k(B)$ in $\mathcal{D}_n^{++}$

- given $d(\cdot,\cdot)$, the natural choice for $\Lambda_k(B)$ is [Alyani et al., 2016]

$$\Lambda_k(B) = \underset{\Lambda \in \mathcal{D}_n^{++}}{\text{argmin}} \; d(BC_kB^T, \Lambda)$$

- the joint diagonalizer B is defined as

$$\underset{B}{\text{argmin}} \; \sum_k w_k d(BC_kB^T, \Lambda_k(B))$$

many choices for the divergence $d(\cdot,\cdot)$

Approximate joint diagonalization (AJD)

- Frobenius distance: least-squares criterion

  AJD in [Cardoso and Souloumiac, 1993]

$$\delta_{\mathsf{F}}^2(C,\Lambda) = \|C-\Lambda\|_{\mathsf{F}}^2 \qquad \Lambda = \mathrm{ddiag}(C)$$

- Kullback-Leibler divergence: from statistics and signal processing

$$d_{\mathsf{KL}}(P,S) = \mathrm{tr}(P^{-1}S - I_n) - \log\det(P^{-1}S)$$

  - left measure - log-likelihood criterion AJD in [Pham, 2000]

$$d_{\mathsf{lKL}}(C,\Lambda) = d_{\mathsf{KL}}(\Lambda,C) \qquad \Lambda = \mathrm{ddiag}(C)$$

  - right measure

$$d_{\mathsf{rKL}}(C,\Lambda) = d_{\mathsf{KL}}(C,\Lambda) \qquad \Lambda = \mathrm{ddiag}(C^{-1})^{-1}$$

Approximate joint diagonalization (AJD)

- natural Riemannian distance: geodesical distance on $\mathcal{S}_n^{++}$ [Bhatia, 2009]

$$\delta_{\mathsf{R}}^2(C,\Lambda) = \left\| \log(\Lambda^{-1/2} C \Lambda^{-1/2}) \right\|_{\mathsf{F}}^2 \qquad \log(C^{-1}\Lambda) = 0$$

- Bhattacharyya distance: closely related to the natural Riemannian distance, numerically cheaper [Sra, 2013]

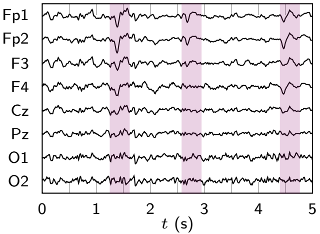
$$\delta_{\mathsf{B}}^2(C,\Lambda) = 4 \log \frac{\det((C+\Lambda)/2)}{\det(C)^{1/2} \det(\Lambda)^{1/2}} \qquad 2 \,\mathrm{ddiag}\left((C+\Lambda)^{-1}\right) = \Lambda^{-1}$$

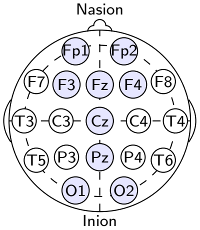
- Wasserstein distance: from optimal transport [Villani, 2008]

$$\delta_{\mathsf{W}}^2(C,\Lambda) = \mathrm{tr}\left(\frac{1}{2}(C+\Lambda) - (\Lambda^{1/2} C \Lambda^{1/2})^{1/2}\right) \quad \mathrm{ddiag}\left((\Lambda^{1/2} C \Lambda^{1/2})^{1/2}\right) = \Lambda$$

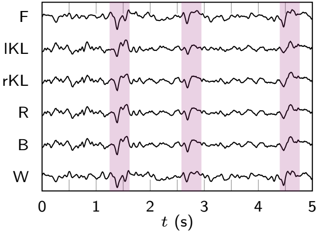
Numerical experiment

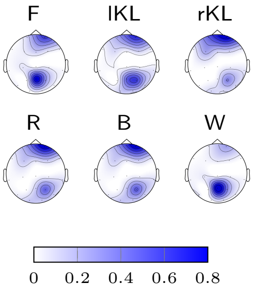
- recording of an epileptic patient - 19 electrodes, sampling rate 128Hz
- goal: retrieve the source corresponding to the 3 peak-slow wave complexes





Numerical experiment





Left: waveforms of the estimated source corresponding to the peak-slow wave complexes for all divergences considered

Right: spatial distributions of the estimated source on the scalp for all divergences considered

Conclusions and perspectives

- different criteria give different information $\rightarrow$ combine them
- try different combinations of divergence / target matrices
- study the theoretical properties of the criteria
- study the links between AJD and centers of mass

Thank you for your attention !

- PhD: October 2015 - September 2018
- Publications:
  - F. Bouchard, L. Korczowski, J. Malick, M. Congedo. *Approximate joint diagonalization within the Riemannian geometry framework*. 24th European Signal Processing Conference (EUSIPCO-2016).
  - F. Bouchard, J. Malick, M. Congedo. *Approximate joint diagonalization according to the natural Riemannian distance*. 13th International Conference on Latent Variable Analysis and Signal Separation (LVA/ICA-2017)
  - F. Bouchard, P. Rodrigues, J. Malick, M. Congedo. *Réduction de dimension pour la séparation aveugle de sources*. Submitted to GRETSI 2017.
  - F. Bouchard, J. Malick, M. Congedo. *Riemannian optimization and approximate joint diagonalization for blind source separation*. Submitted to IEEE Transactions on signal processing.

Alyani, K., Congedo, M., and Moakher, M. (2016).
Diagonality measures of Hermitian positive-definite matrices with application to the approximate joint diagonalization problem.
Linear Algebra and its Applications.

Bhatia, R. (2009).
Positive definite matrices.
Princeton University Press.

Cardoso, J.-F. and Souloumiac, A. (1993).
Blind beamforming for non Gaussian signals.
IEE Proceedings-F, 140(6):362–370.

Comon, P. and Jutten, C. (2010).
Handbook of Blind Source Separation: Independent Component Analysis and Applications.
Academic Press, 1st edition.

Nunez, P. L. and Srinivasan, R. (2006).
Electric fields of the brain: the neurophysics of EEG.
Oxford University Press, USA.

Pham, D.-T. (2000).
Joint approximate diagonalization of positive definite Hermitian matrices.
SIAM J. Matrix Anal. Appl., 22(4):1136–1152.

Sra, S. (2013).
Positive definite matrices and the S-divergence.
arXiv preprint arXiv:1110.1773.

Villani, C. (2008).
Optimal transport: old and new, volume 338.
Springer Science & Business Media.